SOL HW 1.4 p2

September 25, 2016 1:13 PM

Math 8 Honours Assignment 1.4 Prime Factorization and Factors Part 2:

1. Find the lowest value of N such that the square root will become a positive integer:

a) $\sqrt{2^3 5^1 7^2 N}$

$\sqrt{2^2 \cdot 2^1 \times 5^1 \times 7^2 \times N}$

$\sqrt{2^2 \times 7^2} \times \sqrt{2 \times 5 \times N}$

$2 \times 7 \times \sqrt{2 \times 5 \times N}$

$\boxed{N = 10}$

b) $\sqrt{4^2 7^2 5^2 N}$

$\sqrt{4^2 \times 7^2 \times 5^2 \times N}$

$4 \times 7 \times 5 \times \sqrt{N}$

$\boxed{N = 1}$

c) $\sqrt{3^4 5^3 12 N}$

$\sqrt{3^4 \times 5^2 \times 5 \times 2^2 \times 3^1 \times N}$

$3^2 \times 5 \times 2 \times \sqrt{5 \times 3 \times N}$

$N = 15$

d) $\sqrt{38412N}$

$4 \overline{)38412} = 9603$; $9 \overline{)9603} = 1067$; $11 \overline{)1067} = 97$

$\sqrt{4 \times 9 \times 11 \times 97 \times N}$

$2 \times 3 \times \sqrt{11 \times 97 \times N}$

$\boxed{N = 1067}$

e) $\sqrt{13992N}$

$8 \overline{)13992} = 1749$; $3 \overline{)1749} = 583$; $11 \overline{)583} = 53$

$\sqrt{8 \times 3 \times 11 \times 53 \times N}$

$N = 2 \times 3 \times 11 \times 53$

$N = 1749 \times 2$

$\boxed{N = 3498}$

f) $\sqrt{664(N-1)}$

$4 \overline{)664} = 166$; $2 \overline{)166} = 83$

$\sqrt{4 \times 166(N-1)}$

$N - 1 = 166$

$\boxed{N = 167}$

2. Find the lowest value of N such that the cube root will become a positive integer:

a. $\sqrt[3]{2^3 5^1 7^2 N}$

$\sqrt[3]{2^3 \times 5^1 \times 7^2 \times (5^2 \times 7)}$

$\therefore N = 5^2 \times 7$

$= 175$

b. $\sqrt[3]{4^2 7^2 5^2 N}$

$\sqrt[3]{2^4 \times 7^2 \times 5^2 \times N}$

$N = 2^2 \times 7 \times 5$

$= 140$

c. $\sqrt[3]{3^4 5^3 12 N}$

$\sqrt[3]{3^3 \times 3 \times 5^3 \times 2^2 \times 3 \times N}$

$\sqrt[3]{3^3 \times 5^3 \times 2^2 \times 3^2 \times N}$

$N = 2 \times 3$

$= 6$

3. Indicate the number of factors for each of the following numbers:

a) $N = 2^3 3^5$

$2^0, 2^1, 2^2, 2^3$ and $5^0, 5^1, 5^2, 5^3, 5^4, 5^5$

4×6

$= 24$ factors

b) $N = 2^3 3^4 (25)$

$2^3 \times 3^4 \times 5^2$

$(3+1)(4+1)(2+1)$

$4 \times 5 \times 3$

$= 60$

c) $N = 3888$

$= 16 \times 243$

$8 \overline{)3888} = 486$; $2 \overline{)486} = 243$; $3 \overline{)243}$ = 81

$2^4 \times 3^5$

# of factors

$(4+1)(5+1)$

$= 30$

4. Find the lowest value of N such that the integer will have the indicated the indicated number of factors:

a) $2^3 3^N$ $(8\ factors)$ b) $(8)\times 27N$ (48 factors) c) $2^3 3^4 N^2$ $(56\ factors)$

a)
$8 = 4\times 2$
$2^3\times 3^1$
$N = 1$

b)
$2^3\times 3^3\times N.$
$48 = 4\times 4\times 3$
$2^3\times 3^3\times 5^2$
$\boxed{N=25}$ //

c)
$2^3\times 3^4\times (N)^2$
$56 = 8\times 7$
$2^7\times 3^6 \rightarrow N^2 = (2^2\times 3^1)^2$
$\boxed{N=12}$ //

5. Find the sum of all the factors for each of the following:

a) 144 $= 12\times 12$ $= 2^2\times 3\times 2^2\times 3$ $= 2^4\times 3^2$ Sum $= (2^0+2^1+2^2+2^3+2^4)\times(3^0+3^1+3^2)$ $(2^5-1)\left(\frac{3^3-1}{2}\right)$ $= 31\times 13 = 403$ /	b) 7920 $= 792\times 10$ $= 99\times 8\times 10$ $= 3^2\times 11\times 2^4\times 5^1$ Sum $= (2^0+2^1+2^2+2^3+2^4)\times(3^0+3^1+3^2)(11+1)(5+1)$ $= 31\times(13)(12)(6)$ $= 29{,}016$ // $403\times 72 = 806 + 2821 = 29016$	c) $2^3\times 3^2\times 5^3$ Sum $= (2^0+2^1+2^2+2^3)\times(3^0+3^1+3^2)\times(5^0+5^1+5^2+5^3)$ $= (2^4-1)\left(\frac{3^3-1}{2}\right)\left(\frac{5^4-1}{4}\right)$ $= (15)(13)(156)$ $= (195)(156)$ $= 30420$ 195×156: $1170 + 975 + 195 = 30420$

6. What is the largest prime factor of 3045?

$3045 \div 15 = 203$

$3045 = 15\times 203$

LARGEST PRIME FACTOR $= 203$.

7. Find n, such that $2^3 3^2 n = 10!$

$\cancel{2^3}\times\cancel{3^2}\times n = 1\times 2\times 3\times 4\times 5\times 6\times 7\times \cancel{8}\times\cancel{9}\times 10$

$n = \cancel{2}\times \underline{3\times 4}\times\cancel{5}\times\underline{6\times 7}\times\cancel{10}$

$= 100\times 12\times 42$

$= 22400$ //

$42\times 12 = 84 + 14\cdot 10 = 224$

8. How many factors of 4000 are perfect squares?

$4000 = 4\times 1000$
$= 2^2\times 2^3\times 5^3$
$= 2^5\times 5^3$

# of P.S. FACTORS $=$
2^0, 2^2, 2^4 $\times$ 5^0, 5^2

$3\times 2 = 6$ //

9. How many factors of 21,600 are perfect squares?

216×100

$6^3 \times 10^2 = 2^3 \times 3^3 \times 2^2 \times 5^2$

$= 2^5 \times 3^3 \times 5^2$

2^0	3^0	5^0
2^2	3^2	5^2
2^4		

$3 \times 2 \times 2 = 12$ factors.

10. What is the least positive integer that is not a factor of $7!$?

11

11. How many positive integral factors does N have if $N = 6^2 \times 15$?

12. What is the smallest positive integer by which 80 can be multiplied so that the product will be a perfect cube?

13. What is the smallest number that has 1 to 10 as its factors?

14. What is the smallest positive integer that has the numbers 1 to 20 as its factors?

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

17 18 19 20

$$\boxed{N = 2^4 \times 3^2 \times 5 \times 7^1 \times 11^1 \times 13^1 \times 17^1 \times 19^1 =}$$

15. What is the smallest number with 36 factors?

$2 \times 3 \times 2 \times 3 = 36$

$2^2 \times 3^2 \times 5^1 \times 7^1 = 1260$

$4 \times 3 \times 3$

$2^3 \times 3^2 \times 5^2 = 1800$

16. What is the sum of three greatest consecutive integers less than 200 for which the least number has 4 as a factor, the next has 5 as a factor, and the largest number has 6 as its factor?

$(x-1) + (x) + (x+1) \leq 200$

64 65 66

17. What is the smallest positive integer n, for which 88 is a factor of $n!$?

18. Two positive integers have a GCF of $2 \times 3 \times 5$ and a LCM of $2^3 \times 3^4 \times 5 \times 7$. If one of the numbers is 210, find the other number.

19. Find the smallest number N, such that $2^3 3^4 N^2$ has 56 factors.

$\# = 2^3 \times 3^4 \times N^2$ $\quad N = 15 \quad$ 8×7

$$\boxed{2^3\ 3^4\ (3 \times 5)^2}$$

$$2^3 \times 3^4 \times 3^2 \times 5^2$$

$$= 2^3 \times 3^6 \times 5^2$$

$$= 2^3 \times 3^4 \times (2^2 \times 3)^2 \quad N = 12$$

$$= 2^7 \times 3^6$$

20. Two numbers are "*relatively prime*" if they do not share any common factors other than 1. How many positive integers less than or equal to 40 are relatively prime to 40?

$\underline{1}$, $\cancel{2}$, 3, $\cancel{4}$, $\cancel{5}$, $\cancel{6}$, 7, $\cancel{8}$, 9, $\cancel{10}$

11, $\cancel{12}$, 13, $\cancel{14}$, $\cancel{15}$, $\cancel{16}$, 17, $\cancel{18}$, 19, $\cancel{20}$

21, $\cancel{22}$, 23, $\cancel{24}$, $\cancel{25}$, $\cancel{26}$, 27, $\cancel{28}$, 29, $\cancel{30}$

31, $\cancel{32}$, 33, $\cancel{34}$, $\cancel{35}$, $\cancel{36}$, 37, $\cancel{38}$, 39, $\cancel{40}$

$$40 = 8 \times 5$$
$$= 2^3 \times 5$$

21. Challenge: Suppose there are 1000 lockers and 1000 people. The first person opens all the lockers; the second person closes every second locker; the third person changes the state of every third locker [ie: if it's open, he closes it or if it's closed, he opens it]. This process continues, where the nth person changes the state of every nth locker. After all 1000 people have gone through, how many lockers are open?

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓
3	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗	✓	✓	✓
4	✓	✗	✗	✓	✓	✓	✓	✓	✗	✗	✓	✗	✓	✗	✗	✓	✓	✓	✓
5																			